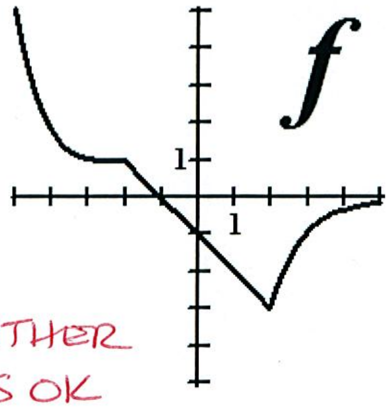
Let $g(x) = \int_1^x f(t)\,dt$, where f is the function whose graph is shown on the right. **SCORE: _____ / 5 PTS**



[a] Find $g'(2)$. **Explain your answer very briefly.**

$\underbrace{g'(2) = f(2)}_{\frac{1}{2}} = \underbrace{-3}_{\frac{1}{2}}$

[b] Find all intervals over which g is concave up. **Explain your answer very briefly.**

$\underbrace{g'(x) = f(x) \text{ INCREASING}}_{1}$ ON $(2, \infty)$ OR $(2, 5)$ 1 EITHER IS OK

[c] Find all intervals over which g is increasing. **Explain your answer very briefly.**

$\underbrace{g'(x) = f(x) > 0}_{1}$ ON $(-\infty, -1)$ OR $(-5, -1)$ 1 EITHER IS OK

A new office building is renting office space month-by-month for $R(x)$ dollars per square meter if the tenant rents x square meters. What is the meaning of the equation $\int_{3000}^{3200} R(x)\,dx = 2000$ in this situation ? **SCORE: _____ / 3 PTS**

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NOTES: **Your answer must use all three numbers from the equation, along with correct units.**
Your answer should NOT use "x", "R(x)", "integral", "antiderivative", "rate of change" or "derivative".

IF YOU ENLARGE YOUR OFFICE FROM $3000\,m^2$ TO $3200\,m^2$, YOUR RENT WILL INCREASE BY \$2000

AJ was looking at BJ's homework. **SCORE: _____ / 3 PTS**
AJ said that BJ's work (shown below) was wrong, but BJ insisted that it was correct. Who was right, and why ?

$$\int_0^{\pi} \sec^2\theta\,d\theta = \tan\theta\Big|_0^{\pi} = \tan\pi - \tan 0 = 0 - 0 = 0$$

GRADED BY ME

AJ — $\sec^2\theta$ IS NOT CONTINUOUS ON $[0, \pi]$ @ $\theta = \frac{\pi}{2}$

If $p(x) = \int_{\sinh^{-1} x}^{e^{2x}} \sin^6 t \, dt$, find $p'(x)$.

SCORE: _____ / 4 PTS

$$\frac{d}{dx}\left[\int_{\sinh^{-1}x}^{0} \sin^6 t\,dt + \int_0^{e^{2x}} \sin^6 t\,dt\right]$$

$$= \frac{d}{dx}\left[-\int_0^{\sinh^{-1}x} \sin^6 t\,dt + \int_0^{e^{2x}} \sin^6 t\,dt\right] \quad 1$$

$$= \frac{-d}{d(\sinh^{-1}x)}\int_0^{\sinh^{-1}x} \sin^6 t\,dt \cdot \frac{d(\sinh^{-1}x)}{dx} + \frac{d}{d(e^{2x})}\int_0^{e^{2x}} \sin^6 t\,dt \cdot \frac{d(e^{2x})}{dx}$$

$$= -\frac{1}{\sqrt{1+x^2}} \sin^6 \sinh^{-1} x + 2e^{2x} \sin^6 e^{2x}$$

1, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$

Evaluate the following integrals.

ALL PARTS WORTH $\frac{1}{2}$ POINT EXCEPT THOSE MARKED EXPLICITLY

SCORE: _____ / 10 PTS

[a] $\int \frac{(2+t)^2}{\sqrt[3]{t}} dt$

$$= \int \frac{4+4t+t^2}{t^{\frac{1}{3}}} dt$$

$$= \int \left(4t^{-\frac{1}{3}} + 4t^{\frac{2}{3}} + t^{\frac{5}{3}}\right) dt \quad 1$$

$$= 6t^{\frac{2}{3}} + \frac{12}{5}t^{\frac{5}{3}} + \frac{3}{8}t^{\frac{8}{3}} + C$$

[b] $\int_{-1}^{1} \frac{7x^3}{(3-2x^6)^5} dx$

$f(x)$ IS CONTINUOUS ON $[-1,1]$

$$f(-x) = \frac{7(-x)^3}{(3-2(-x)^6)^5} = -\frac{7x^3}{(3-2x^6)^5} \quad 1$$

$$= -f(x)$$

SO $f(x)$ IS ODD ON $[-1,1]$

SO $\int_{-1}^{1} f(x)\,dx = 0$

[c] $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{(\pi - \arctan 2y)^2}{1+4y^2} dy$

$u = \pi - \arctan 2y$

$$du = -\frac{2}{1+4y^2} dy \rightarrow -\frac{1}{2} du = \frac{1}{1+4y^2} dy$$

$$y = \frac{1}{2} \rightarrow u = \pi - \arctan 1 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$y = -\frac{1}{2} \rightarrow u = \pi - \arctan(-1) = \pi - -\frac{\pi}{4} = \frac{5\pi}{4}$$

$$\int_{\frac{5\pi}{4}}^{\frac{3\pi}{4}} -\frac{1}{2}u^2\,du = -\frac{1}{6}u^3\Big|_{\frac{5\pi}{4}}^{\frac{3\pi}{4}} = -\frac{1}{6}\left(\left(\frac{3\pi}{4}\right)^3 - \left(\frac{5\pi}{4}\right)^3\right)$$

1

MUST HAVE "du"

$$= -\frac{1}{6}\left(\frac{27\pi^3}{64} - \frac{125\pi^3}{64}\right) = \frac{-1}{\cancel{6}_3} \cdot \frac{\cancel{-98}^{49}\pi^3}{64} = \frac{49\pi^3}{192} \quad 1$$